647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2024 (Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one out of two. (07)

(1) State and prove Hausdorff property.

(2) X is a metric space. Show that intersection of finite number of open subsets of X is open. Explain with example that this statement is not true for infinite number of open subsets of X.

Que-1(B) Answer any one out of two. (04)

(1) Define d on $\mathbb{R}$ as $d(x,y) = \begin{cases} 0 & ; \ x = y \\ \sqrt{x^2+y^2} & ; x \neq y \end{cases}$.

Then show that ($\mathbb{R}$,d) is a metric space.

(2) (i) Let $\mathbb{R}$ has discrete metric. Find N(2,0.2) in $\mathbb{R}$.

(ii) Find E^0 where E={2,4,6,8,…………..}.

(iii) Construct set of real numbers which has exactly three limit points.

Que-1(C) Answer any one out of two. (03)

(1) Show that Cantor set is closed and prove that $\frac{9}{10} \in C$.

(2) Check whether sets $\mathbb{N}, \mathbb{Q}$ and $\mathbb{R}$ are closed and/or open.

Que-2(A) Answer any one out of two. (07)

(1) prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall$ ε>0 there exist partition P of [a,b] such that

U(P,f) - L(P,f) < ε , Where $\|P\| < \delta$.

(2) Show that product of two bounded R-integrable functions is R-integrable.

Que-2(B) Answer any one out of two. (04)

(1) Find $\int_0^1 e^x \, dx$ using definition of R-integration.

(2) If bounded function f on [a,b] is monotonic then show that f is R-integrable.

Que-2(C) Answer any one out of two. (03)

(1) Using Darboux theorem find value of

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots\ldots\ldots\ldots + \left(\frac{n}{n}\right)^2\right]$$

(2) Let F(x)=m where m$\in \mathbb{R}$ for all x $\in [a,b]$ then prove that $F(x) \in \mathbb{R}_{[a,b]}$

Que-3(A) Answer any one out of two. (07)

(1) If bounded function f is R-integrable in [a,b] and F(x)= $\int_a^x f(t)dt$, Where a≤x≤b then show that F(x) is continuous in [a,b].

(2) State and prove general form of first mean value theorem of integral calculus.

Que-3(B) Answer any one out of two. (04)

(1) Show that $\int_0^{\frac{\pi}{4}} x\tan(x)dx < \frac{\pi}{8}\log_e 2$.

(2) Show that set of all 4^{th} roots of 1 forms a group under multiplication.

Que-3(C) Answer any one out of two. (03)

(1) Let G = $\mathbb{R}$-{-1} and define binary operation * on G as, $a*b=a+b+ab$, for $a,b\in G$ then show that (G,*) is group and find 2^{-1} in G.

(2) Verify general form of first mean value theorem by taking $f(x)=x$ and $g(x)=e^x$ where $x\in[-1,1]$.

Que-4(A) Answer any one out of two. (07)

(1) G is finite group and $H\leq G$. Show that $o(H) \mid o(G)$.

(2) Prove that order of permutation $f\in S_n$ is L.C.M. of length of disjoint cycles of f.

Que-4(B) Answer any one out of two. (04)

(1) $G=\{(a,b)/a,b\in \mathbb{R}$ and $a\neq 0\}$ and binary operation * define on G as: For $a,b,c,d \in \mathbb{R}$, $(a,b)*(c,d)=(ac,bc+d)$. Show that if $H=\{(1,b)/b\in \mathbb{R}\}$ then $H\leq G$. Where (G,*) is group.

(2) Define even and odd permutation. $\sigma\epsilon S_6$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 2 & 1 & 6 & 5 \end{pmatrix}$ then check whether σ is even or odd.

Que-4(C) Answer any one out of two. (03)

(1) G is group and $H\leq G$. then prove that any two left coset of H in G is either equal or disjoint.

(2) Prove that $(Z_3,+_3)$ is group.

Que-5(A) Answer any one out of two. (07)

(1) State and prove Cayley's theorem.

(2) G is finite group and $a \epsilon G$. prove that $o(a) \mid o(G)$.

Que-5(B) Answer any one out of two. (04)

(1) G is group and $H\leq G$. If index of H in G is 2 then prove that H is normal subgroup of G.

(2) Prepare group table in usual notation for quotient group S_3/A_3.

Que-5(C) Answer any one out of two. (03)

(1) For $n\geq 2$ show that A_n is normal subgroup of S_n.

(2) Prove that for finite cyclic group, order of group is same as order of its generator.

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